{B.Sc Part I (Physics Hons)
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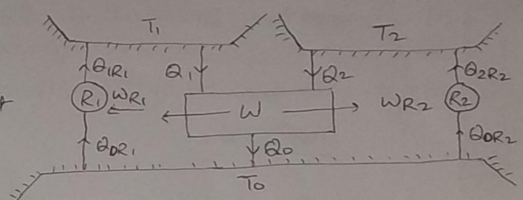
Question:- Deduce clausius inequality relation and show that it hold goods for any cyclic process.?

Ans:- **Clausius inequality:-** clausius principle states that" The ratio of the amount of heat transferred by a large number of heat reservoier to their temperatures is always less than or equal to Zero" Mathematically

$$\int \frac{dQ}{T} \leq 0$$

Proof:- Let us consider a closed system such as one shown in figure. where a heat Engine E and two carnot's (reverisible) refrigerators R_1 and R_2 operate between two hot reservoier or Sources A and B at temp T_1 and T_2 respectively and a common cold reservoier or sink C at temp T_0

Let us suppose that Engine E absorbs a quantity of heat Q_1 from a Source A and at Temp T_1 and a quantity of heat Q_2 from a Source B at temp. T_2. It does W units of work and reject a quantity of heat Q_0 to the sink C at temperature T_0.



Again, let us suppose that the refrugerators R_1 and R_2 absorbs quantities of heat Q_{OR_1} and Q_{OR_2} from the sink C, have W_{R_1} and W_{R_2} units of work done upon them and reject ~~and Q~~ Q_{OR_1} and Q_{OR_2} quantities of heat to revervoirs A and B respectively.

If the heat absorbed from and rejected to reservoirs A and B be the same, then we have

$$Q_{1R_1} = Q_1 \text{ and } Q_{2R_2} = Q_2$$

Clearly with the heat engine and two refrigerators operating simultaneously, reservoirs A and B remain unaffected. The only possible change in the system may be either that the heat absorbed from the sink $Q_{OR_1} + Q_{OR_2}$ may not be equal to the heat rejected to it, i.e Q_0 or that the work done on the refrigerators may not be equal to the work W done by the engine E. Thus the sink may gain or loss heat and the system may gain or or loss work.

If the sink loses heat and the system gains equivalent amout of work, it would mean that all the heat absorbed from the sink is converted into work. It is perfectly all right according to the first law of thermodynamics due to it merely isnists on equivalence of heat and work. However the Second law firmly denies this possibilities of conversion of the whole of the heat absorbed from the sink into work. This means, therefore that $Q_0 > (Q_{OR_1} + Q_{OR_2})$ except in the limiting case when the entire process is reversible and $Q_0 = (Q_{OR_1} + Q_{OR_2})$.

Now, in the case of refrigerator $R_1 = \frac{Q_{1R_1}}{T_1} = \frac{Q_{OR_1}}{T_0}$ ...... (1)

and in case of refrigerator $R_2 = \frac{Q_{2R_2}}{T_2} = \frac{Q_{OR_2}}{T_0}$ ...... (2)

Conventionally heat rejected or given out by a body or a reservoier is reckoned positive and heat absoobed by it is negative so that Q_1, Q_2, Q_{OR_1} & Q_{OR_2} are (+ve) while Q_0, Q_{1R_1} and Q_{2R_2} are (-ve). Ignoring the signs, we may treat them all as (+ve) quantities. Mathematically, we have from (1)

$$\frac{Q_{OR_1}}{T_0} = \frac{Q_{1R_1}}{T_1}$$

or, $Q_{OR_1} = \left(\frac{Q_{1R_1}}{T_1}\right) \times T_0$ and from (2) $Q_{OR_2} = \left(\frac{Q_{2R_2}}{T_2}\right) \cdot T_0$

Since $Q_{1R_1} = Q_1$ and $Q_{2R_2} = Q_2$

$\therefore Q_{OR_1} = \left(\frac{Q_1}{T_1}\right) T_0$ .... (3)

and $Q_{OR_2} = \left(\frac{Q_2}{T_2}\right) T_0$ .... (4)

Hence the net amount of heats drawn from the sink $= Q_{OR_1} + Q_{OR_2} + Q_0$. Since in accordance with the 2nd law, this must be less than 0 or in the limiting case 0. Thus we have $(Q_{OR} + Q_{OR} + Q_0) \leq 0$

or. $\left(\frac{Q_{1R_1}}{T_1}\right) T_0 + \left(\frac{Q_{2R_1}}{T_2}\right) T_0 + Q_0 \leq 0$

Deviding by T_0 throughout, we have $\frac{Q_{1R_1}}{T_1} + \frac{Q_2R_2}{T_2} + \frac{Q_0}{T_0} \leq 0$

or in general $\sum \frac{Q}{T} \leq 0$ .... (5)

If the number of reservoiers be infinite and change occurs in infinitesimally small steps we may write eqn (5) as

$$\oint \frac{\delta Q}{T} \leq 0 \quad \cdots \quad (6)$$

Relations (5) and (6) represents clausius inequality, where equally holds good for reversible process and inequality for irreversible process. This relation holds good for any cyclic process whether reversible or irreversible.

It is thus clear that the direction of irreversible processes in a closed system is always such that the entropy of the system and its ~~surro~~ surroundings or the entropy of the universe always increases. In the limiting case of a reversible closed system, the entropy remains constant, but in no close system it is found to diminish by processes of any kind.